Climate Variability and Climate Change in Light of Physics: A mini-review

Ayesha khurshid1(c), Zehra Hashmi2, Saba Javaid3

Abstract:

A thermodynamic equilibrium system is forced, dissipative, nonlinear, complex, and heterogeneous. The system is influenced by many external factors, both natural and anthropogenic, at many scales of motion in time and space. In this review, we present observations of climate phenomena and the governing equations for planetary-scale flow. An overview of the hierarchy of models used in climate science is presented. A combination of dynamical systems theory, on one hand, and nonequilibrium statistical physics, on the other hand, is shown for the first time in this paper to help explain and predict the behavior of a system. The complementary points of view allow self-consistent consideration of subgrid-scale phenomena as stochastic processes, as well as the unified analysis of natural climate variability and forced climate change, in addition to the significance of climate sensitivity, response, and predictability.

Key Words: Climate Change, The hierarchy of models, Nonequilibrium statistical physics, Dissipative, Anthropogenic, Planetary-scale

1 Introduction to climate dynamic:

Due to humanity's inability to collect data of consistent quality, sufficient regional resolution, and acceptable temporal coverage, the climate sciences face a basic issue. In instrumental datasets, there are significant challenges with synchronic and diachronic coherence. Furthermore, such databases only go back around one to two centuries at most. We'll start with instrumental datasets and then go on to historical and proxy datasets, which rely on indirect evidence to determine the value of meteorological observables prior to the industrial revolution. In the nineteenth century, meteorological stations were created in Europe and North America. Since then, the network of observations has grown in size and quality, as has the technology underpinning data collecting and storage. Nonetheless, the geographical density of data varies drastically around the world at any one time, with much sparser observations over the ocean and overland areas typified by low population density or technical development; for example, [15], Figure 1.

- Ayesha Khurshid from NED University of engineering and Technology, Pakistan,. E-mail: aisha.khurshid91@gmail.com
- Zehra Hashmi from NED University of engineering and Technology, Pakistan,. E-mail: zehra.12hashmi@gmail.com
- Saba Javaid from NED University of engineering and Technology, Pakistan,. E-mail: hars4@yahoo.com

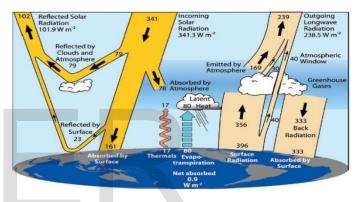


Figure 1: Energy fluxes in the Earth system (W m2) on a global scale. The fluxes on the left represent solar visible and ultraviolet radiation, the fluxes on the right represent terrestrial infrared radiation, and the fluxes in the centre indicate nonradiative fluxes. Trenberth, Fasullo, and Kiehl published Trenberth, Fasullo, and Kiehl in 2009.

In the late 1960s, the introduction of polar-orbiting and geostationary satellites ushered in a revolution in the collection of meteorological, land surface, and ocean surface data. Many climatic variables are now being remotely sensed from the most remote parts of the globe; for example, they measure the overall intensity and spectral features of emitted infrared and reflected visible and ultraviolet radiation, and complex algorithms link their raw measurements to actual atmospheric properties like temperature and cloud cover. Figure 2 depicts the evolution of the schematically observational network for climatic data, whereas Figure 3 depicts the evolution of the observational network for climatic data. The WMO's Global Observing System, which organises the gathering and quality control of weather and climate data across the globe, depicts the equipment that make up the Global Observing System today.

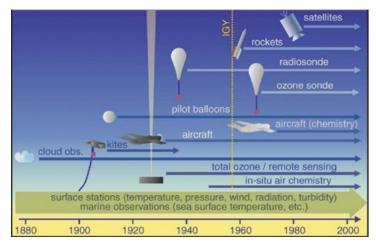


Figure 2. Schematic diagram representing the evolution of the observing network for weather and climate data. The dotted vertical line corresponds to the International Geophysical Year (IGY). Courtesy of Dick Dee.

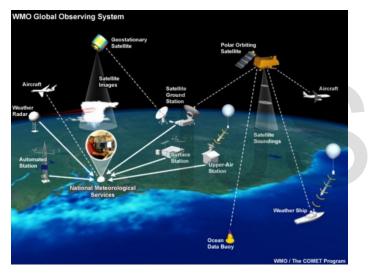


Figure 3. An illustration of the instruments and platforms that compose the World Meteorological Organization's (WMO's) Global Observing System (GOS). From the COMET website9 of the University Corporation for Atmospheric Research (UCAR), sponsored in part through a cooperative agreement with the National Oceanic and Atmospheric Administration (NOAA), U.S. Department of Commerce (DOC).

The article is outlined as follows. In Section 1, reviewed papers related to climate change are described through collecting various type of data from the articles and papers of famous researchers who have done study on the topic of climate change and its effects. Next, Section 2, in which the detailed study is presented to shed the light on basic facts of climate change, in this section climate model system and their types are also discussed to get the clear insight of the study title.

To identify related contributions, different digital databases are utilized to collect important literature. The query for papers containing is "climate change [7]. There are several steps applied to filter and select important articles on the topic. The first step is removing duplicated publications among digital databases. Next, the titles and abstracts of the publications were checked to examine the relevancy of the selected articles. Then, full-text reading to find out whether they are appropriate to be involved in the final set.

2 Literature work:

Tradition depicts that the study of weather patterns in the midlatitudes, particularly during the boreal winter when winds are fiercest and variability is greatest, is the focus of dynamics meteorology. In this part, we attempt to enhance the reader's comprehension of large-scale atmospheric dynamics by emphasizing a component of the atmosphere that is rather well known at this time. According to what can be seen, a broad variety of procedures have an impact on the spectral properties of the data. It is well documented that the weather in the midatitudes is closely tied to synoptic disturbances that last 3-10 days and cover a spatial span of 1000–2000 km in length [19]. They develop in a similar way to the well-known eastward-spreading cyclones and anticyclones, as a result of baroclinic instability converting the available potential energy in the zonal flow into eddy kinetic energy. In order for the Lorenz energy cycle to function properly, this conversion must take place whenever the centre of mass of an unstable atmospheric system is reduced. When temperature gradients and vertical wind shear are significant enough, they may cause baroclinic instability [3,34]. These conditions are more easily demonstrated during the winter season, when the equator and the poles are separated by a large temperature difference and a vigorous mid-latitude jet, which are both present at the time. [18] was a pioneer in the field of space-time spectrum analysis, which was founded by Pratt and Fraedrich and Bottger. Spectral power is also attributed to each range of geographical scales and time intervals, in addition to information on air eddy direction and speed, which is included in the analysis. To reconstruct the propagation of air waves, one must first do a Fourier analysis on a onedimensional spatial field in order to determine their frequency. Current study is being conducted on the dynamics and energizing of planetary waves. It is intrinsically linked to the descriptions and explanations of several highly nonlinear properties of the atmosphere including blocking events, which are defined as persistent, large-scale departures from zonally symmetric general circulation [13,1,35] shown that a blocking occurrence may be separated from the more typical zonal flow event. Consistent blocking events have the potential to have a longterm impact on the weather in large areas of the world. To just one example, [23] have shown that large-scale flows and the related synoptic-scale weather may be forecast for periods longer than the usual limit of 10–15 days. For as long as the model's starting state is within the range of possibility, the most sophisticated and highly resolved NWP models may reasonably correctly forecast whether or not a blocking event would continue, but they are unable to predict when a blocking event will begin or collapse [10]. More simplistic climate models, on the other hand, are unable to effectively recreate the spatiotemporal statistics of such occurrences; in fact, very little progress has been made in this area in the recent two decades [7]. The findings of this work reveal that the blocking event dynamics and statistics seen in the field by [35] may be replicated in the laboratory.

2.1.1 Basic facts of climate sciences

Positive and negative feedback mechanisms, instabilities, and saturation processes all work together to produce a chaotic, out-of-control system in which the climate is always shifting and unpredictable. Processes of this kind may occur across a wide range of geographical and temporal dimensions, and they can include a variety of chemical and physical species as well. The system's diverse phenomenology includes boundary layers in both the atmosphere and the ocean, as well as turbulence at various sizes around the globe [17]. Large-scale agents that drive and modify the development of the system, including differential solar heating, the Earth's rotation and gravity, also affect the system's behaviour. It's not rare for complex physics to be combined with chaotic dynamics, as is generally the case with chaotic systems.

Furthermore, the climate system's inherent unpredictability over long time periods is substantially influenced by even the tiniest changes in human and natural forcing sources [14,26,29].

The phenomenology of the climate system is frequently studied from a variety of perspectives that are mutually reinforcing. These are some examples:

- Waves such as Rossby and equatorially restricted waves [12] are critical components in the transmission of energy and water vapour from the atmosphere to the seas.
- It is possible to see a variety of particle-like phenomena throughout the world's weather systems: hurricanes, extra tropical cyclones, and marine vortices are just a few examples. A significant impact on the local characteristics of the climate system, as well as the qualities of its subsystems and sub domains [27,31].
- A turbulent cascade is required for the geostrophic turbulence process [4], as well as for mixing and dissipation of the planetary boundary layer [4,36].

Each of these points of view has elements of overlap and complementarity that are mutually advantageous to both parties [17,26].

However, while significant progress has been made in this area [17], it remains challenging to explain and forecast climate dynamics as a result of the following additional barriers, which are inherent in any complex nonlinear system that is out of equilibrium:

- Each subsystem, such as the atmosphere, the seas, and the cryosphere, has its own set of physical and chemical characteristics, as well as its own time and space scales.
- Using a complicated set of techniques, each of these subsystems communicates with the others.
- Temperatures are constantly being influenced by changes in solar radiation and natural and humanmade activities that alter the composition of the atmosphere.
- Adaptation of model reduction techniques to the absence of scale separation between processes, which results in extensive parameterization of subgrid-size processes in numerical models, must be undertaken.
- It is difficult to replicate previous temperatures before to the Industrial Revolution because there aren't enough long-term climate data that are correct, homogeneous, and high-resolution; and, finally, there aren't enough long-term climate data that are accurate, homogeneous, and highresolution.
- In order to comprehend how climate change happens, we must first grasp what we don't know. We can only know what we don't know at any particular moment in time. As a consequence, in the absence of time-dependent elements, it is difficult to distinguish between the climate system's reaction to a range of external events and its inherent unpredictability in the climate system. To provide just one example, as [22] shown, it is difficult to discern between the consequences of climate change resulting from natural variability and those resulting from human involvement [26,25].

2.1.2 Climate system models

A climate model is a mathematical representation of the climate system that is based on physical, biological, and chemical concepts (Fig.4). The equations arising from these rules are very intricate, and as a result, they must be solved numerically.

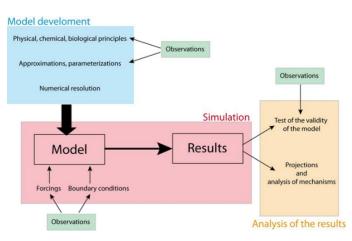


Figure 4: Schematic representation of the development and use of a climate model.

Several climate models have been constructed in order to simulate and grasp the effects of greenhouse gas and aerosol emissions on the climate. These models are used to perform climate predictions. Models may prove to be important tools in our quest to get a better understanding of the climate system and its causes. It is undeniable that climatologists are unable to conduct experiments on the actual climate system in order to determine the precise involvement of a particular process or to test a certain hypothesis. This, on the other hand, can be accomplished in the virtual world of climate models. When dealing with highly nonlinear systems, it is critical that the study design be rigorous in order to get the best findings possible. In a basic investigation, it is possible to leave out a process or an element in order to get a preliminary estimate of the function of that process or element. If the effect of growing CO₂ concentration on radiative characteristics of the atmosphere is not taken into consideration, for example, this may be avoided.

3 Types of models

The degree of complexity of the processes represented by a model may also be used to distinguish between different models (Fig. 3.2). Generic circulation models (GCMs), as a general rule, are designed to capture as much information as possible about a system's internal dynamics. A GCM was formed as an acronym for global climate models (GCM) since one of its primary objectives is to correctly simulate the three-dimensional structure of winds and currents. The AGCMs (atmospheric) and the OGCMs (oceanic) were traditionally classified into two divisions, respectively (OGCMs). Acronyms such as AOGCM (Atmosphere Ocean General Circulation Model) and CGCM (Coupled General Circulation Model) are often used in the context of climate research.

In contrast, fundamental climate models provide a much simplified form of the climate system's dynamical system. The variables are averaged across large areas, generally the whole Earth, and as a consequence, a significant number of processes are left out of the equation. Because of this, the adaptability of EBMs is severely stifled. EMICs are those that fall in the centre of the spectrum (Earth Models of Intermediate Complexity). Even though GCMs are a more complicated model of the system than EBMs, they have been reduced and parameterized to account for numerous processes that are expressly included into GCMs. The EMIC model category is the most inclusive of all the models. GCMs may be compared to basic models, and some GCMs are degraded versions of basic models.

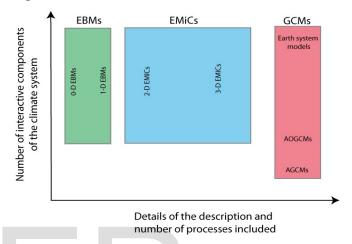


Figure 4.1: Types of climate model.

3.1 Effects of climate change

Plants' growth and development, on the other hand, may be negatively affected by very cold or hot conditions. Growing temperatures have been associated with early blooming and maturity in a broad range of crops during the past few decades [6]. Cranfurd and Wheeler are two of the most wellknown names in American history (2009a). Increases in maximum temperatures may have a negative impact on crop production and reproductive success, among other things (climate or weather). A day in which the temperature exceeds 30 degrees Celsius when the weather is dry may result in a 1.7 percent reduction in corn production. According to [24,5] conducted a study in Ghanaian communities and discovered that severe weather events had an influence on rural food production, transportation, processing, and storage, as well as on the overall economy. Farm-based storage facilities should be expanded, connectivity should be improved, particularly feeder roads that connect food-producing regions with important markets, farmers should be provided with early warning systems, farmers should be provided with loans, and supplemental irrigation should be implemented to improve food security in this region. It is possible that certain individuals and ethnic groups will be less able to adjust to changes in the food system as a result of cultural norms that prohibit the use of specific foods.

Climate change has an influence on human health in both direct and indirect ways, according to researchers. Extreme

heat has a particularly negative impact on the elderly, who are especially sensitive . There is a great deal of evidence to suggest [27].

3.2 Future climate changes

According to the multi-model average, the rise in global mean temperature by 2010 is associated with warming in all places. Worldwide warming is associated with an increase in global precipitation. An increased capacity to store water in the atmosphere, as a consequence of the Clausius-Clapeyron Equation, is seen on warmer planets, which results in increased evaporation over the oceans. It is estimated that growth in 2100 would be somewhere between 1 percent and 8 percent more than it was in the late twentieth century, depending on whatever model and scenario is used. Furthermore, this rise in precipitation is not uniformly distributed throughout the country and exhibits significant seasonal variability. A rise in winter and summer precipitation at high latitudes is predicted by both the multimodel mean and the overwhelming majority of individual models, according to the latest available data. Precipitation is also expected to increase across tropical seas and in areas impacted by the South Asian summer monsoons, according to the forecast. Tropical Central America and the Caribbean are expected to get significant rainfall, as will the subtropics and the Mediterranean region in general.

3.4 Assessing progress in climate change

In order to combat climate change, governments must submit Nationally Determined Contributions (NDCs) to the Paris Agreement, which set national targets for 2030 and are due by December 31, 2018. Global climate change mitigation (i.e., reducing greenhouse gas emissions) is included in all of the original NDCs announced in 2015; in addition, 131 NDCs contain policies and/or programs aimed at climate change adaptation and enhancing resilience. The early 2015 NDC reports collectively fell short of the temperature goals set out in the Paris Climate Agreement. In the unlikely event that all non-binding international agreements are adhered to, the global average temperature is anticipated to rise by 3.2°C by the end of the century (Environment, Letter from the executive director in Review 2019). Countries are urged to draught Nationally Determined Contributions (NDCs) that become increasingly ambitious over time in order to help bridge the "emissions gap" [33]. In 2020, it is predicted that the United Nations Framework Convention on Climate Change (UNFCCC) will issue its first official proposal to increase the ambition of Nationally Determined Contributions (NDCs). There have already been reactions from a large number of countries. In 2019, 103 nations committed to increasing their Nationally Determined Contributions (NDCs) by 2020, accounting for 38.4 percent of global greenhouse gas emissions at the time of agreement (Cat climate target update tracker 2020). Nationally Determined Contributions (NDCs) submitted to the United

Nations Framework Convention on Climate Change by November 2020 represented just 4.6 percent of global GHG emissions, and not all of the submissions were backed up by concrete actions. It will be necessary for large emitters to dramatically increase their mitigation efforts if the world is to keep global temperature rise well below 2°C.

4 Conclusion:

The purpose of this study was to emphasise some of the key physical and mathematical components that can aid in the description, comprehension, and prediction of climate variability and change. Observational, theoretical, and numerical elements have all been taken into account. Dynamical systems theory and nonequilibrium statistical mechanics have been used extensively. We attempted to give a unified view of the climate system's temporal dependency, multiscale character, and metastability. The complicated relationship between inherent climatic variability and the climate's reaction to disturbances has been highlighted. The talk also attempted to demonstrate how important mathematical and physical methods may aid in the resolution of the major difficulties confronting climate scientists. These problems will not be solved simply by raising the resolution of numerical models and including additional physical and biogeochemical processes into them.

References:

- [1] Benzi, R., Malguzzi, P., Speranza, A., & Sutera, A. (1986). The statistical properties of general atmospheric circulation: Observational evidence and a minimal theory of bimodality. *Quarterly Journal of the Royal Meteorological Society*, 112(473), 661–674. <u>https://doi.org/10.1002/qi.49711247306</u>.
- [2] Cat climate target update tracker. Climate Action Tracker. (2020). Retrieved March 20, 2022, from <u>https://climateactiontracker.org/climate-target-update-tracker/</u>.
- [3] Charney, J. G. (1947). The dynamics of long waves in a baroclinic westerly current. Journal of Meteorology, 4(5), 136–162.
 <u>https://doi.org/10.1175/1520-</u> 0460(1947)004%lt/0126ttdolwii&rgt/2.0.co;2

0469(1947)004<0136:tdolwi>2.0.co;2

- [4] Charney, J. G. (1971). Geostrophic turbulence. Journal of the Atmospheric Sciences, 28(6), 1087– 1095. <u>https://doi.org/10.1175/1520-0469(1971)028<1087:gt>2.0.co;2</u>
- [5] Codjoe, S. N., & Owusu, G. (2011). Climate change/variability and food systems: Evidence from the Afram Plains, Ghana. *Regional Environmental Change*, 11(4), 753–765. <u>https://doi.org/10.1007/s10113-011-0211-3</u>.
- [6] Craufurd, P. Q., & Wheeler, T. R. (2009). Climate change and the flowering time of annual crops.

Journal of Experimental Botany, 60(9), 2529–2539. https://doi.org/10.1093/jxb/erp196.

- [7] D'Andrea, F., & Davini, P. (2020). Northern Hemisphere atmospheric blocking simulation in present and future climate. <u>https://doi.org/10.5194/egusphere-egu2020-18294</u>.
- [8] Environment, U. N. (n.d.). UNEP Annual Report: Letter from the executive director - 2019 in Review. UNEP. Retrieved March 20, 2022, from <u>https://www.unep.org/annualreport/2019/index</u> .php#:~:text=UNEP%20organized%20the%20World d%20Environment,WHO%20and%20the%20World %20Bank.
- [9] Environment, U. N. (2019). Letter from the executive director: 2019 in Review. UNEP. Retrieved March 20, 2022, from <u>https://www.unep.org/resources/annual-</u> <u>report/letter-executive-director-2019-review.</u>
- [10] Ferranti, L., Corti, S., & Janousek, M. (2014). Flowdependent verification of the ECMWF ensemble over the Euro-Atlantic Sector. *Quarterly Journal of the Royal Meteorological Society*, 141(688), 916–924. <u>https://doi.org/10.1002/qj.2411</u>.
- [11] Fraedrich, K., & Böttger, H. (1978). A wavenumber-frequency analysis of the 500 MB geopotential at 50°N. *Journal of the Atmospheric Sciences*, 35(4), 745–750. <u>https://doi.org/10.1175/1520-0469(1978)035<0745:awfaot>2.0.co;2</u>
- [12] Ghil, M. (1976). Climate stability for a sellers-type model. *Journal of the Atmospheric Sciences*, 33(1), 3–20. <u>https://doi.org/10.1175/1520-0469(1976)033<0003:csfast>2.0.co;2.</u>
- [13] Ghil, M., & Childress, S. (1987). Topics in geophysical fluid dynamics: Atmospheric Dynamics, dynamo theory, and Climate Dynamics. *Applied Mathematical Sciences*. <u>https://doi.org/10.1007/978-1-4612-1052-8</u>.
- [14] Ghil, M., Mullhaupt, A., & Pestiaux, P. (1987). Deep Water Formation and Quaternary glaciations. *Climate Dynamics*, 2(1), 1-10. <u>https://doi.org/10.1007/bf01088850</u>.
- [15] Ghil, M., & Malanotte-Rizzoli, P. (1991). Data assimilation in meteorology and Oceanography. *Advances in Geophysics Volume* 33, 141–266. <u>https://doi.org/10.1016/s0065-2687(08)60442-2</u>
- [16] Ghil, M., & Robertson, A. W. (2002). "waves" vs. "particles" in the atmosphere's phase space: A pathway to long-range forecasting? *Proceedings of the National Academy of Sciences*, 99(suppl_1), 2493– 2500. <u>https://doi.org/10.1073/pnas.012580899</u>.
- [17] Ghil, M. (2019). A century of nonlinearity in the Geosciences. *Earth and Space Science*, 6(7), 1007–1042. <u>https://doi.org/10.1029/2019ea000599</u>.
- [18] Hayashi, Y. (1971). A generalized method of resolving disturbances into progressive and

retrogressive waves by space fourier and time cross-spectral analyses. Journal of the Meteorological Society of Japan. Ser. II, 49(2),

125–128.

<u>5</u>.

https://doi.org/10.2151/jmsj1965.49.2_12

- [19] Holton, J. R., & Hakim, G. J. (2013). Introduction. An Introduction to Dynamic Meteorology, 1– 29. <u>https://doi.org/10.1016/b978-0-12-384866-</u> <u>6.00001-5</u>.
- [20] Lorenz, E. N. (1955). Available potential energy and the maintenance of the general circulation. *Tellus*, 7(2), 157–167. <u>https://doi.org/10.1111/j.2153-3490.1955.tb01148.x</u>.
- [21] Lorenz, E. N. (1967). The nature and theory of the general circulation of the atmosphere. (SELBSTVERL.).
- [22] Lorenz, E. N. (1979). Forced and free variations of weather and climate. *Journal of the Atmospheric Sciences*, 36(8), 1367–1376. <a href="https://doi.org/10.1175/1520-0469(1979)036<1367:fafvow>2.0.co;2">https://doi.org/10.1175/1520-0469(1979)036<1367:fafvow>2.0.co;2.
- [23] Lorenz (1996) predictability partly solved. (n.d.). Retrieved March 20, 2022, from <u>https://eapsweb.mit.edu/sites/default/files/misc</u> <u>lorenz_1996_predictability_problem_partly_solve</u> <u>d.pdf</u>.
- [24] Lobell, D. B., Schlenker, W., & Costa-Roberts, J. (2011). Climate trends and global crop production since 1980. *Science*, 333(6042), 616–620. https://doi.org/10.1126/science.1204531.
- [25] Lucarini, V., & Sarno, S. (2011). A statistical mechanical approach for the computation of the climatic response to general Forcings. *Nonlinear Processes in Geophysics*, 18(1), 7–28. https://doi.org/10.5194/npg-18-7-2011.
- [26] Lucarini, V., Blender, R., Herbert, C., Ragone, F., Pascale, S., & Wouters, J. (2014). Mathematical and physical ideas for climate science. *Reviews of Geophysics*, 52(4), 809–859. https://doi.org/10.1002/2013rg000446.
- [27] McMichael, A. J., Woodruff, R. E., & Hales, S. (2006). Climate change and human health: Present and future risks. *The Lancet*, 367(9513), 859-869. <u>https://doi.org/10.1016/s0140-6736(06)68079-3</u>.
- [28] McWilliams, J. C. (2019). A perspective on the legacy of Edward Lorenz. *Earth and Space Science*, 6(3), 336-350. https://doi.org/10.1029/2018ea000434.
- [29] Peixoto José P., & Oort, A. H. (1992). Physics of climate. Amazon. Retrieved March 20, 2022, from <u>https://www.amazon.com/Physics-Climate-Jose-P-Peixoto/dp/0883187124</u>.
- [30] Pratt, R. W. (1976). The interpretation of space-time spectral quantities. *Journal of the Atmospheric Sciences*, 33(6), 1060–1066.

https://doi.org/10.1175/1520-0469(1976)033<1060:tiosts>2.0.co;2.

- [31] Salmon, R. (1998). Lectures on geophysical fluid dynamics. <u>https://doi.org/10.1093/oso/9780195108088.001.0</u> 001.
- [32] Speranza, A. (1983). Deterministic and statistical properties of the Westerlies. *Pure and Applied Geophysics PAGEOPH*, 121(3), 511-562. https://doi.org/10.1007/bf02590154.
- [33] UNFCCC Article 4 commitments. (2015). Retrieved March 20, 2022, from <u>https://unfccc.int/files/cooperation_and_support</u> <u>/ldc/application/pdf/article4.pdf</u>.

[34] Vallis, G. K. (2017). Atmospheric and Oceanic Fluid Dynamics.

https://doi.org/10.1017/9781107588417.

- [35] Weeks, E. R., Tian, Y., Urbach, J. S., Ide, K., Swinney, H. L., & Ghil, M. (1997). Transitions between blocked and zonal flows in a rotating annulus with topography. *Science*, 278(5343), 1598–1601. <u>https://doi.org/10.1126/science.278.5343.1598</u>.
- [36] Zilitinkevich, S. S. (1975). Resistance laws and prediction equations for the depth of the planetary boundary layer. *Journal of the Atmospheric Sciences*, 32(4), 741-752. <u>https://doi.org/10.1175/1520-0469(1975)032<0741:rlapef>2.0.co;2</u>.

IJSER